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RELATIVISTIC TRANSFORMATION OF THE TEMPERATURE AND OF
CERTAIN OTHER THERMODYNAMIC QUANTITIES

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RELATIVISTIC TRANSFORMATION OF THE TEMPERATURE AND OF
CERTAIN OTHER THERMODYNAMIC QUANTITIES *

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by H. Arzelies

SUMMARY

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The present memoir has for its object to show that the formulas thus far admitted, are unacceptable, and that it is appropriate to construct the relativistic thermodynamics on new foundations. It is specified in particular, that the temperature is to be transformed according to the formula $T = T_0 / \sqrt{1 - \beta^2}$ and not $T = T_0 \sqrt{1 - \beta^2}$.

Author

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1. - INTRODUCTION

Most of relativistic authors, and in particular von Laue [1], Pauli [2], Tolman [3], Louis de Broglie [4], Moller [5], McCrea [6], give the following formulas for the transition from the proper referential K_0 (in which the considered thermodynamic system is at rest) to another, Galilean referential K (velocity v relative to K_0)

$$U = \frac{U_0 + \beta^2 p_0 v_0}{\sqrt{1 - \beta^2}},$$

$$U + p v = \frac{U_0 + p_0 v_0}{\sqrt{1 - \beta^2}},$$

$$d\mathcal{F} = d\mathcal{F}_0 \sqrt{1 - \beta^2} - \frac{\beta^2}{\sqrt{1 - \beta^2}} d(U_0 + p_0 v_0),$$

$$dQ = \sqrt{1 - \beta^2} dQ_0,$$

$$S = S_0, \quad T = T_0 \sqrt{1 - \beta^2}$$

* Transformation relativiste de la température et de quelques autres grandeurs thermodynamiques.

U denoting the internal energy, \mathcal{V} the volume, p the uniform pressure which the system is assumed to undergo, \mathcal{F} is the work supplied by the system outside, Q the quantity of absorbed heat, S the entropy, T the temperature.

Others make the distinction between the Planck heat and the covariant heat, but they too transform the Planck heat and temperature according to the above relations. This viewpoint will be found in the Costa de Beauregard work [7] with bibliography.

I intend to show that, except for the transformation of S , these formulas are erroneous. I shall first of all establish the formulas which I consider to be correct; I shall then show for what precise reason the formulas, subject to criticism, are not acceptable (relativity of the simultaneity).

2. - TRANSFORMATION OF THE WORK OF A UNIFORM PRESSURE ACTING UPON A SOLID

2.1.- Pressure Parallel to the Velocity. - Let us consider, at rest within K_0 , a rectangular parallelepiped, of which the two opposed bases of surface dS_0 , are subject to a uniform pressure p_0 between the moments of time $t_0 = 0$ and $t_0 = T_0$. There will be no possible confusion with the same letters, denoting in other paragraphs the temperature.

Let us denote these two bases by the letters A and B, and let l_0 be their distance for $t_0 = 0$. If the parallelepiped surrounds an elastic medium at the instant $t_0 = T_0$, the faces will have shifted by Δl_0 . The work supplied between 0 and t_0 has for value, in K_0

$$d\mathcal{F}_0 = 2p_0 dS_0 \Delta l_0 = p_0 \Delta \mathcal{V}_0.$$

The impulsion supplied is zero.

Let us relate the phenomenon to a referential K ,

relative to which the parallelepiped has the velocity \mathcal{V} , normal to faces A and B (we shall subsequently consider the other case). The only clear

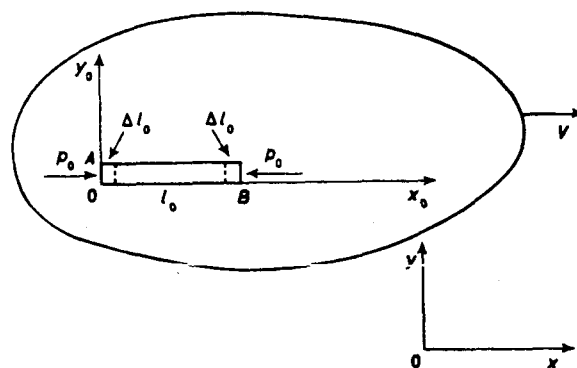


Fig. 1.

and correct procedure consists in transforming the coordinates of the punctual events. Here the events are as follows : (Fig. 1):

the pressure begins in A	$x_0 = 0$	$t_0 = 0$,
" " " in B	$x_0 = l_0$	$t_0 = 0$,
the pressure ends in A	$x_0 = \Delta l_0$	$t_0 = T_0$,
" " " in B	$x_0 = l_0 - \Delta l_0$	$t_0 = T_0$.

Let us transform these coordinates with the formulas

$$x = \frac{x_0 + vt_0}{\sqrt{1 - \beta^2}} \quad t = \frac{t_0 + (v/c^2)x_0}{\sqrt{1 - \beta^2}}.$$

In K they become :

the pressure begins in A	$x = 0$,	$t = 0$,
" " " in B	$x = \frac{l_0}{\sqrt{1 - \beta^2}}$,	$t = \frac{(v/c^2)l_0}{\sqrt{1 - \beta^2}}$,
the pressure ends in A	$x = \frac{\Delta l_0 + vT_0}{\sqrt{1 - \beta^2}}$,	$t = \frac{T_0 + (v/c^2)\Delta l_0}{\sqrt{1 - \beta^2}}$,
" " " in B	$x = \frac{l_0 - \Delta l_0 + vT_0}{\sqrt{1 - \beta^2}}$,	$t = \frac{T_0 + (v/c^2)(l_0 - \Delta l_0)}{\sqrt{1 - \beta^2}}$.

Let us compute this work. In order to transform the forces, and since here, within K_0 , the point of application shifts, we must utilize the formulas

$$X = X_0 + \frac{(vu_{0x}/c^2)Y_0 + (v/c^2)u_{0z}Z_0}{1 + vu_{0x}/c^2}$$

$$Y = \frac{\sqrt{1 - v^2/c^2}}{1 + vu_{0x}/c^2} Y_0, \quad Z = \frac{\sqrt{1 - v^2/c^2}}{1 + vu_{0x}/c^2} Z_0,$$

denoting by u_0 the velocity of the point of applying of forces. In the present case

$$u_{0x} = u_{0z} = 0, \quad Y_0 = Z_0 = 0,$$

$$X = X_0, \quad Y = Z = 0.$$

Thus, the force which exerts itself upon the faces A and B has in K the value $p_0 S_0$. Over the face A we have the work

$$d\mathcal{F}_A = p_0 dS_0 \frac{\Delta l_0 + v T_0}{\sqrt{1 - \beta^2}}$$

and over B

$$d\mathcal{F}_B = - p_0 dS_0 \frac{-\Delta l_0 + v T_0}{\sqrt{1 - \beta^2}}.$$

The total work in K is

$$d\mathcal{F} = \frac{2p_0 dS_0 \Delta l_0}{\sqrt{1 - \beta^2}} = \frac{p_0 \Delta \mathcal{V}_0}{\sqrt{1 - \beta^2}}.$$

Thus we obtain the transformation relation

$$d\mathcal{F} = \frac{d\mathcal{F}_0}{\sqrt{1 - \beta^2}}.$$

Let us now provide for the intervention of the volumes \mathcal{V} and $\mathcal{V} - d\mathcal{V}$ which correspond by simultaneous observation in K to the volumes \mathcal{V}_0 and $\mathcal{V}_0 - \Delta \mathcal{V}_0$. We then have

$$d\mathcal{F} = \frac{p d\mathcal{V}}{1 - \beta^2}$$

and not $p d\mathcal{V}$, as is often written.

We shall further analyze the phenomenon. The body is at rest in K_0 which is in agreement with a resultant force and impulsion equal to zero. In K the solid has a uniform velocity. But, because of energy inertia, its mass increases and the quantity of motion undergoes thus the accretion

$$v \frac{d\mathcal{F}}{c^2} = \frac{2v p_0 dS_0 \Delta l_0}{c^2 \sqrt{1 - \beta^2}}.$$

To that effect an impulse of equal value is required in the direction of \mathcal{V} . Indeed, the force acting upon the face A provides the impulse

$$p_0 dS_0 \frac{T_0 + (v/c^2) \Delta l_0}{\sqrt{1 - \beta^2}}$$

$$\begin{array}{lll} \text{the pressure ends in } A & x = \frac{vT_0}{\sqrt{1-\beta^2}}, & y = \Delta l_0, \quad t = \frac{T_0}{\sqrt{1-\beta^2}}, \\ \text{" " " in } B & x = \frac{vT_0}{\sqrt{1-\beta^2}}, & y = l_0 - \Delta l_0, \quad t = \frac{T_0}{\sqrt{1-\beta^2}}. \end{array}$$

Let us compute this work. In the transformation formulas of the force components, we have here

$$u_{0x} = u_{0z} = 0, \quad X_0 = Z_0 = 0,$$

$$u_{0y} = \frac{\Delta l_0}{T_0}$$

and subsequently

$$X = \frac{v}{c^2} \frac{\Delta l_0}{T_0} Y_0, \quad Y = Y_0 \sqrt{1-\beta^2}.$$

The forces normal to the velocity supply the work

$$2Y_0 \sqrt{1-\beta^2} \Delta l_0 = 2p_0 dS_0 \sqrt{1-\beta^2} \Delta l_0 = p_0 \Delta \mathcal{V}_0 \sqrt{1-\beta^2}.$$

The forces parallel to the velocity provide the work

$$\frac{2v}{c^2} \frac{\Delta l_0}{T_0} Y_0 \frac{vT_0}{\sqrt{1-\beta^2}} = \frac{2v^2}{c^2} \Delta l_0 \frac{p_0 dS_0}{\sqrt{1-\beta^2}} = p_0 \Delta \mathcal{V}_0 \frac{\beta^2}{\sqrt{1-\beta^2}}.$$

Hence, the total work is

$$d\mathcal{F} = p_0 \Delta \mathcal{V}_0 \left(\sqrt{1-\beta^2} + \frac{\beta^2}{\sqrt{1-\beta^2}} \right) = \frac{p_0 \Delta \mathcal{V}_0}{\sqrt{1-\beta^2}}$$

and finally

$$d\mathcal{F} = \frac{d\mathcal{F}_0}{\sqrt{1-\beta^2}} = \frac{p \Delta \mathcal{V}}{1-\beta^2}.$$

We should note the entirely remarkable role played in K by the component of the force parallel to the velocity (component that does not exist in K_0).

Let us consider now the question of impulses. The quantity of motion of a solid in the state of uniform velocity undergoes the accretion

$$v \frac{d\mathcal{F}}{c^2} = v \frac{d\mathcal{F}_0}{c^2 \sqrt{1-\beta^2}}.$$

An equal impulse is required in the direction of \mathbf{v} .

But we have a force X that acts in A and B during the time $T_0/\sqrt{1-\beta^2}$ and hence the impulse

$$\frac{2v \Delta l_0}{c^2} p_0 ds_0 \sqrt{1-\beta^2} \frac{T_0}{\sqrt{1-\beta^2}} = \frac{v}{c^2} \frac{p_0 \Delta \mathcal{V}_0}{\sqrt{1-\beta^2}} = \frac{v}{c^2} \frac{d\mathcal{F}_0}{\sqrt{1-\beta^2}}.$$

Thus all is in order. Note that the normal impulse to the speed is zero since the same force Y acts during the same time $T_0/\sqrt{1-\beta^2}$ in the opposite directions over the two faces. Therefore, there is neither impulse nor transverse quantity of motion.

3. - TRANSFORMATION OF INTERNAL ENERGY AND OF THE QUANTITY OF HEAT

If W is the total energy, the total mass of the solid is W/c^2 . The quantity W is thus transformed as a mass

$$W = \frac{W_0}{\sqrt{1-\beta^2}}. \quad (1)$$

By virtue of equivalence the quantity of heat is transformed as a work

$$Q = \frac{Q_0}{\sqrt{1-\beta^2}}.$$

At any rate, it can be demonstrated directly starting from the definition of Q . The quantity of heat is a diluted kinetic energy.

Assume that at a given moment of time a group of particles move in a cotic fashion about their inertia center G , fixed in a referential K . The velocity \underline{u} of one of the particles is at the angle α with the axis Ox_0 . With respect to another referential K , the total energy has for expression

$$W = m_0 c^2 \frac{1 + (uv/c^2) \cos \alpha}{\sqrt{1-u^2/c^2} \sqrt{1-v^2/c^2}}.$$

This expression is obtained by writing $W = mc^2$ for K and by transforming the velocity of the particle.

For the aggregate of particles we have the total energy :

$$W = \frac{1}{\sqrt{1-v^2/c^2}} \sum \frac{m_0 c^2}{\sqrt{1-u^2/c^2}} + \frac{v}{\sqrt{1-v^2/c^2}} \sum \frac{m_0 u \cos \alpha}{\sqrt{1-u^2/c^2}}.$$

By virtue of the fixedness of G , we have in K_0

$$\sum \frac{m_0 u_x}{\sqrt{1-u^2/c^2}} = 0, \quad \sum \frac{m_0 u_y}{\sqrt{1-u^2/c^2}} = 0, \quad \sum \frac{m_0 u_z}{\sqrt{1-u^2/c^2}} = 0.$$

Hence results (1). In other respects, if $\sum E_c$ and $\sum E_{0c}$ denote the total kinetic energy,

$$Q_0 = \sum E_{0c}, \quad U_0 = W_0, \\ \dot{Q} = \sum E_c - \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) \sum w_0 c^2, \quad U = W - \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) \sum w_0 c^2.$$

Hence the transformation of Q and U .

4. - ENTROPY AND TEMPERATURE

All the authors show, by referring to its statistical interpretation, that the entropy is an invariant

$$S = S_0.$$

If we wish to preserve an invariant form to the Carnot principle, we must transform the temperature according to the formula

$$T = \frac{T_0}{\sqrt{1-\beta^2}}.$$

This transformation is, at any rate, obtainable directly starting from the definition of the temperature. However, this is not a proper place for that definition to be discussed in the general case. But everyone admits that in case of perfect gases

$$T_0 = k \bar{E}_{0c} \quad T = k \left[\bar{E}_c - m_0 c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) \right].$$

5. - INVARIANT FORM OF THE EQUATION FOR PERFECT GASES

This equation is written in proper referential

$$p_0 \mathcal{V}_0 = RT_0.$$

In another referential, and treating R as a constant, we have

$$\frac{p \mathcal{V}}{1 - \beta^2} = RT.$$

6. - QUADRIVECTORS

With the above transformations, the principle of equivalence has the invariant form

$$dQ = dU - d\mathcal{F}.$$

Let us introduce the following quadrivectors :

quadrivector-force K^i of components

$$K^a = \frac{F^a}{\sqrt{1 - \beta^2}}, \quad K^4 = \frac{i}{c} \frac{d\mathcal{F}}{dt_0}$$

being the force and t_0 the proper time;

quadrivector-impulse Π^i of components

$$\Pi^i = \frac{U_0}{c^2} V^i \quad (V^i \text{ being the quadrivector-velocity})$$

quadrivector-heat

$$Q^i = \frac{Q_0}{c^2} V^i.$$

The equivalence principle is written

$$K^i dt_0 = d\Pi^i - dQ^i.$$

By defining a quadrivector temperature

$$T^i = T_0 V^i$$

the Carnot principle is written

$$dQ^i < dS_0 T^i.$$

7. - REMARK

Remark. - One might also have to involve the densities of heat and entropy and the corresponding quadrivectors

$$q' = q_0 V', \quad \sigma' = \sigma_0 V'.$$

This would impart to the Carnot principle the expressions

$$\partial, q' d\Omega < - i \frac{dS_0}{T_0} T' T',$$

$$\partial, \sigma' > \frac{\partial, q'}{T_0}$$

being the element of volume with four dimensions.

We might also write

$$\partial, \sigma' d\Omega > ic \frac{dQ_0}{T_0}$$

form due to Tolman and which remains valid with the new transformations. But the preceding expressions appear to be more interesting.

8. - CHARACTERISTIC FUNCTIONS OF A MOBILE SYSTEM

They have for definition and for the respective transformation formulas :

enthalpy :

$$H = U + \frac{p\mathcal{V}}{1-\beta^2}, \quad H = \frac{H_0}{\sqrt{1-\beta^2}}$$

free energy :

$$A = U - TS, \quad A = \frac{A_0}{\sqrt{1-\beta^2}}$$

thermodynamic potential or free enthalpy :

$$\Phi = U - TS + \frac{p\mathcal{V}}{1-\beta^2} = H - TS = A + \frac{p\mathcal{V}}{1-\beta^2}, \quad \bar{\Phi} = \frac{\Phi_0}{\sqrt{1-\beta^2}}.$$

9. - BLACK RADIATION

Assume an inclosed space of volume \mathcal{V} containing black radiation and endowed with a velocity \mathcal{V} . The preceding authors give the following

formulas for the total energy W and the impulse I

$$W = a_0 \gamma T^4 \frac{1 + \beta^2/3}{(1 - \beta^2)^3}, \quad I = \frac{4}{3} \frac{a_0 \gamma T^4}{(1 - \beta^2)^3} \frac{v}{c^2}.$$

These formulas are due to Mosengeil; they were adopted by the already named authors (von Laue, Moller, etc.). With my transformations I am led to replace the above expressions by

$$W = (1 - \beta^2) a_0 \gamma T^4 = \frac{W_0}{\sqrt{1 - \beta^2}},$$

$$I = (1 - \beta^2) a_0 \gamma T^4 \frac{v}{c^2}.$$

The coefficient a_0 is defined by the Stefan law

$$U_0 = \gamma_0 a_0 T_0^4.$$

10. - CRITICAL REMARKS.

a) One may see that the divergence between my results and the previous ones originates essentially in the transformation of $d\mathcal{F}$. The preceding authors utilize incorrect formulas for the transformation of the force; they do not take account of the fact that even the proper referential, the point of appliance of the force, is mobile.

At any rate, the new formulas are far more satisfactory. It is indeed natural in relativity that W , Q and T transform themselves as a mass or as an energy.

b) We must beware, however, not to consider my text as rejecting the heat Q_p and the Planck temperature T_p at all in order to adopt the covariant heat Q and temperature T . Indeed, the preceding authors write, for example,

$$dS = \frac{dQ_r}{T_r}, \quad T_r = T_0 \sqrt{1 - \beta^2},$$

$$Q_r \neq Q \quad T_r = T, \quad Q_r = Q_0 \sqrt{1 - \beta^2}$$

whereas I write

$$dS = \frac{dQ_r}{T_r}, \quad T_r = \frac{T_0}{\sqrt{1-\beta^2}},$$

$$Q_r = Q, \quad T_r = T, \quad Q_r = \frac{Q_0}{\sqrt{1-\beta^2}}.$$

Thus, I do not reject one of the categories of concepts, but I show that both categories are identical. Finally, one should not assimilate my temperature T to the temperature $\psi = 1/T$ utilized by certain authors. The utilization of ψ seems to me useless; but if it is used, it must be transformed according to

$$\psi = \psi_0 \sqrt{1-\beta^2}$$

whereas other previous authors write

$$\psi = \frac{\psi_0}{\sqrt{1-\beta^2}}.$$

c) Schlomka [8] states that both, the heat and the temperature are invariants. To show this he considers in K_0 two moving bodies of identical proper mass m_0 , endowed with equal and opposed velocities. Prior to the impact we have in K_0 the impulse and the energy

$$p^0 = 0, \quad W^0 = 2m_0c^2 + W_c^0.$$

In another referential K

$$p = \frac{2m_0 + W/c^2}{\sqrt{1-v^2/c^2}}, \quad W = \frac{2m_0c^2 + W_c^0}{\sqrt{1-v^2/c^2}}.$$

After an inelastic impact the two moving bodies remain at contact; the new proper mass then is M_0 , so that

$$M_0c^2 = 2m_0c^2 + W_c^0.$$

We then may consider that the kinetic energy transformed itself into heat of proper value

$$Q_0 = W_c^0.$$

We verify that the difference of kinetic energies in K , prior and

after the impact, is an invariant, but this difference is not equal to the quantity of heat, as believed by Schlomka.

Indeed, the total energy has for expression

$$\left(2m_0 + \frac{W_0^2}{c^2}\right)\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) + \left(2m_0 + \frac{W_0^2}{c^2}\right)c^2.$$

We must consider as heat in K the sum of the terms containing W_c^0

$$Q = \frac{W_0^2}{\sqrt{1-v^2/c^2}} = \frac{Q_0}{\sqrt{1-v^2/c^2}}.$$

The heat is equal to the difference of the proper masses plus the kinetic energy of that difference. In other words, part of the kinetic energy after the impact constitutes heat in motion.

The Schlomka error consists in not considering the kinetic energy of the heat, but only of energy at rest. In this case one might also say that the mass is an invariant; but this would be language abuse, for with that sort of language any physical quantity could be an invariant.

Thus, the conclusion relative to T is also to be rejected.

**** THE END ****

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REFERENCES

- [1].- M. VON LAUE.- Die Relativitätstheorie (Theory of Relativity), 3rd Ed. Vieweg, 1952.
- [2].- W. PAULI.- Theory of Relativity. London, 1958.
- [3].- R. C. TOLMAN.- Relativity Thermodynamics and Cosmology, Oxford, 1955.
- [4].- L. DE BROGLIE.- Sur la variance relativiste de la température. (On the relativistic variance of temperature).- Cahiers de Physique, June 1948.
- [5].- C. MOLLER.- Theory of Relativity, Oxford, 1955.
- [6].- W. H. MCCREA.- Relativity Physics, New York, 1960.
- [7].- O. COSTA DE BEAUREGARD.- La théorie de relativité restreinte. (The Restricted Theory of Relativity). PARIS, 1949.
- [8].- T. SOCHLOMKA.- Zur Lorentzinvariantz der Wärme und der absoluten Temperatur (On the Lorentz Variant of Heat and of Absolute Temperature).- Phys. Verh. No. 4, p. 161, 1953.
- [9].- H. ARZELIES.- La Cinématique relativiste, Paris 1955.
- [10].- H. ARZELIES.- La dynamique relativiste et ses applications.- (The Relativistic Dynamics and Their Applications).- Paris, 1957.
- H.E. For the transformation of forces see p. 35.
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